**Fluid Mechanics for Civil and Mechanical Engineering**
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**Lecture No 10**
**Conservation of Momentum and its Applications**

Welcome to you this course on a fluid mechanics. It is a very interestingly that today we have a 10th lecture, which is the half way of the fluid mechanics course, what I have been teaching you and today will cover the conservations of momentum and its applications, which is really a interesting subject, in the fluid mechanics using the Reynolds transport theorems and the control volume concept.

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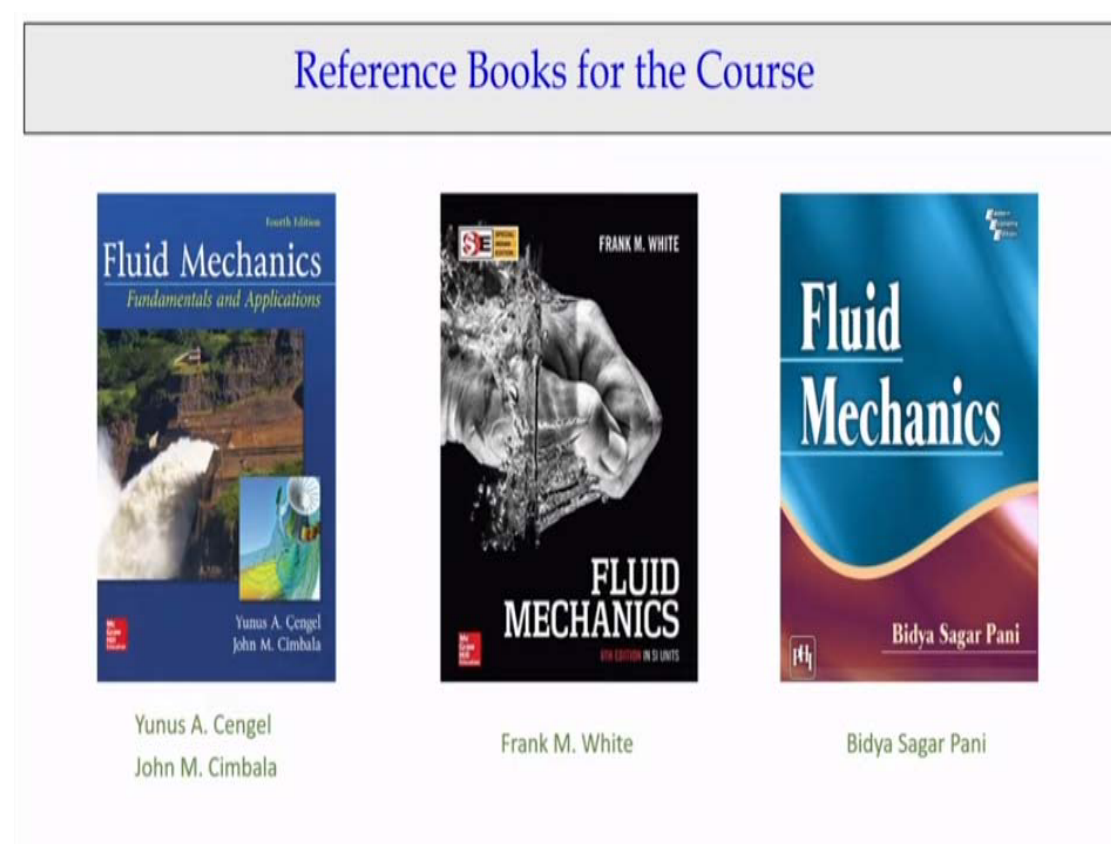


Considering that aspect, I will go through you how we can simplify these Reynold transport theorems for a control volumes and how we can simplify in terms pressure distributions and the velocity distributions. So, how we do the simplifications of these equations and then we apply that equations for, a engineering applications like finding out the force components, the velocity component and the pressure distributions.

As I stated earlier, I just follow the books of these three, the mostly I am talking about a mid-path between fluid mechanics books by FM White, which is big mathematical oriented, whereas if you talk about fluid mechanics fundamental and applications, which is more illustrated oriented. So, I tried to make it to you in between of these mathematics and illustrations oriented and other book as you know it, this fluid mechanics by Professor Bidya

Sagar Pani.

So, let us come back to the recap of the previous lectures. As we said it earlier, the basically the mass conservations equations.

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And we also solved the GATE questions on mass conservation equations. Then we also discuss very details that the Reynolds transport theorem for linear momentum equations and that is what are the two control volumes; one is fixed control volume another for the moving control volume. And that what we talked about the force acting a control volume. There will be two types of forces, body forces and the surface forces. The surface forces are pressures and the reactions forces, that what we talked a lot.

Discuss also today body forces mostly what the problems we can consider is only the gravity force we consider as a body force, not the electric magnetic forces. Then, we also discussed about the momentum flux correction factors, this is a very simplified concept used to determine the momentum flux passing through a non-uniform cross-sections and using this correction factor, momentum flux correction factors. And also I gave you a example of jet experiments, how we conducted in laboratory.
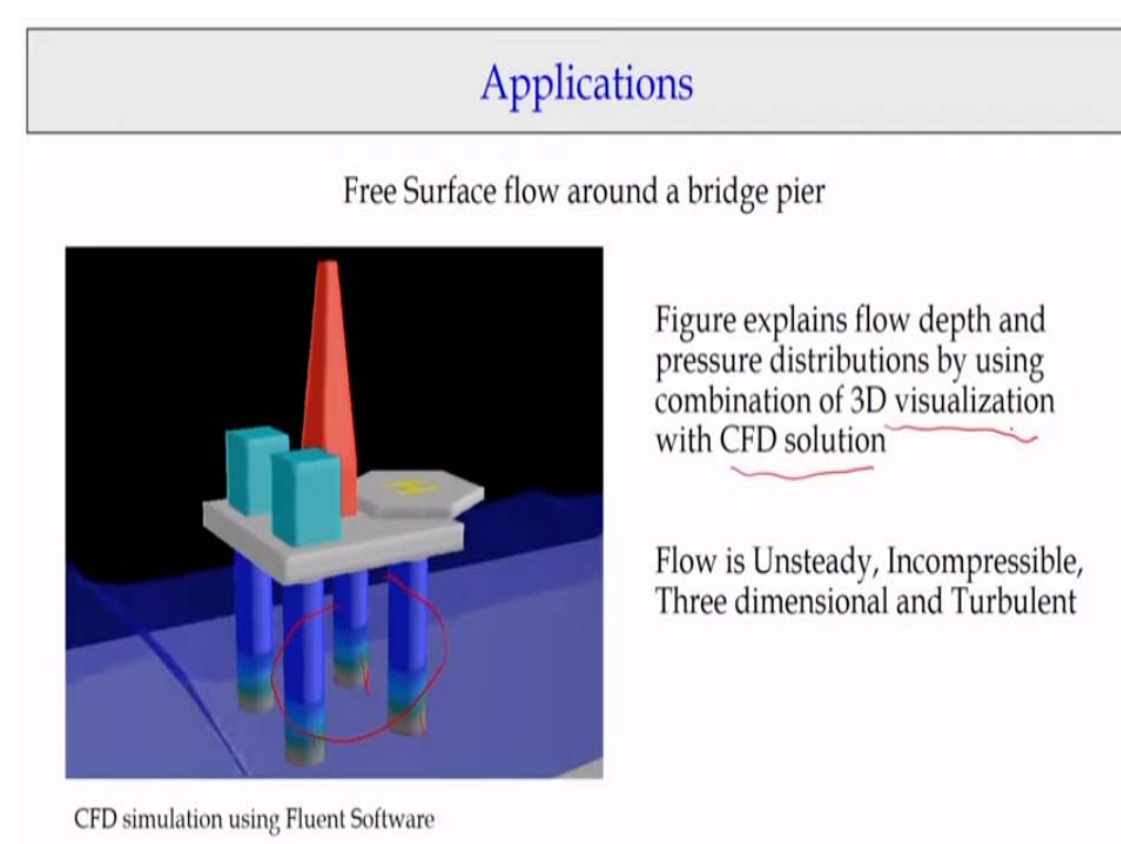
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**Contents of Lecture 10**

Now let us come to today's lecture, will have a steady flow across missions for the linear momentum equations. Many of the times, we have a linear momentum equations we solve for the one inlet or one outlet and in one directions and some of the problems we have solved with no external forces, then how the momentum equations can be simplified. Then fourth what we will talk about, when you apply the linear momentum equations, what are the hints and tips, what should we consider when you apply that linear momentum equations.

Then we will commit to solve for example problems of previous GATE questions, we will solve it, then we will have a summary of today lectures. Now let us come back to very interesting 3D figures what you can see it.

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**Applications**

Free Surface flow around a bridge pier

Figure explains flow depth and pressure distributions by using combination of 3D visualization with CFD solution

Flow is Unsteady, Incompressible, Three dimensional and Turbulent

CFD simulation using Fluent Software

It is, there are bridge piers, there are the bridge piers are here and the flow is coming it, which

is unsteady flow. And if you look at this color fringes, its showing how the pressure diagrams are changing, just you look it, how the 3D view we were getting it, how the flow is passing through a bridge piers and how the pressure distributions are changing it, if you look at these colors of this ones, how the pressure distributions are changing it. This is today's world, is a possible, because the computational fluid dynamic solutions are available as well as the 3D visualizations tools are developed.

So, you can feel like that we have conducted an experiment in a fluent to get it how the pressure distributions, how the flow of the various sensors happening it, in a group of bridge piers are there and how they are interacting each other, if you can look at these 3D figures. If you look it this way, this flow is unsteady, is perish with the time, incompressible flow, 3 dimensional and the turbulent flow. So, all the complex flow we can solve it using the CFD and we can visualize that to in 3D visualizations.

Somebody can feel it, how the flow is going on. If you can look at that, there is a breaking of the waves is also happening it, if you just to look it, there is a breaking of the waves are happening it and how this flow is coming it, how the hitting in different bridge piers and the different bridge piers having the different amount of pressures acting on the force due to the pressure diagrams are the different. So these types of visualizations nowadays it is possible with help of the CFD solution the 3D visualizations.

It is possible nowadays to solve the full fledge turbulent Navier-Stokes equations with some aphorisms, it is possible. So that is why pointed to tell that even if I just talk about the control volume concept, which is a very gross characteristic, but today it is a possible to have a this to 3D visualizes and the CFD solutions which gives us, just a real life problems like what I am showing it, a pre-surface flow around the bridge, the breaking of the waves, and also the pressure distributions are changes from the pier to pier.

All its a possible because of the CFD solutions of have three dimensional turbulent unsteady incompressible flow, that is what is available, also 3D visualizations. With this note, let us go to the our level where we are talking about the integral concept of control volumes what we apply it for a simple problems.

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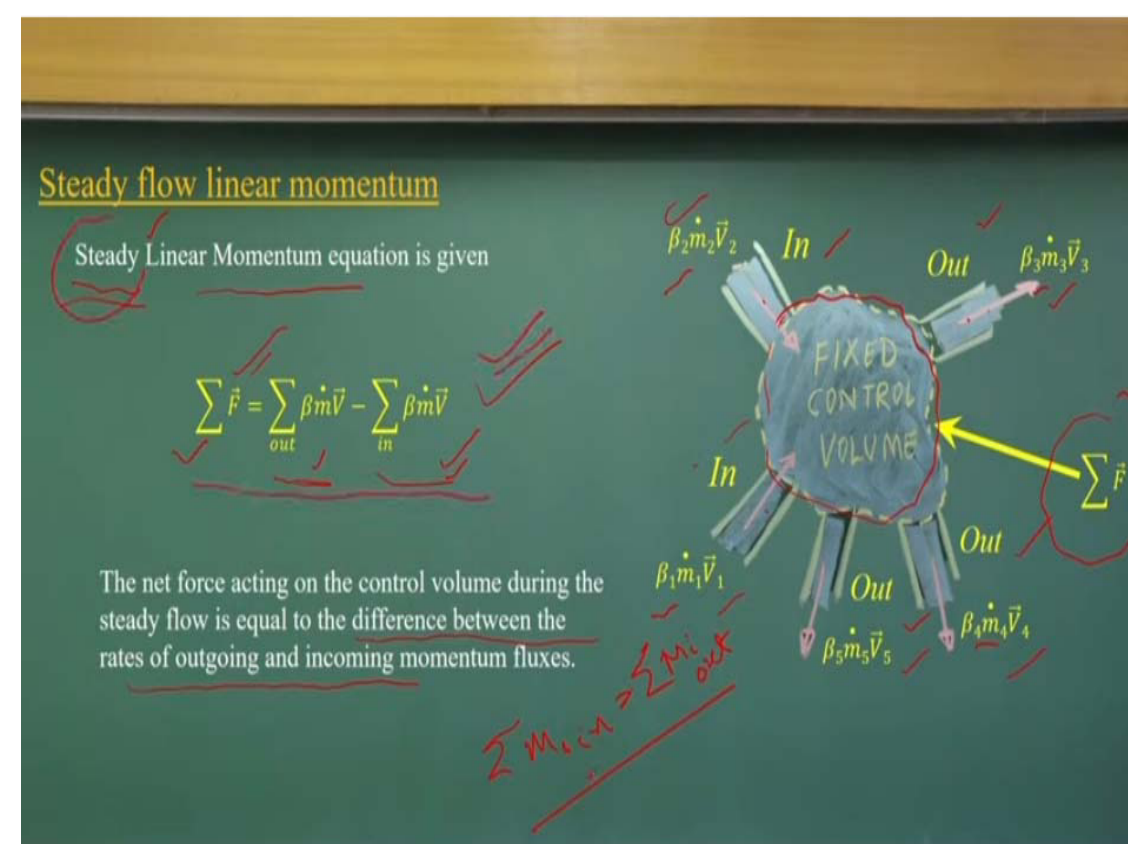
Now, go for a simple problems, where you have a linear momentum equations like you have a fixed control volume, just to look at, you have a fixed control volume. There are the in are there, okay, and there are the outs are three, and this is a fixed control volume. So there is inflows, this is outflows. There is inflows and outflows. The sum of the force will be act on these, if it is a steady linear momentum equations.

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

The time derivative components become 0, so we will have a sum of force is equal to, this is still I am putting in a vector notation is equal to the, in case of steady, the difference between the rate of outgoing and incoming momentum fluxes. So what we will do it, some of the momentum flux, what is coming as an inflow, that the mass flux into the velocity that is a momentum flux, then we have the beta one and beta two, which is representing momentum flux correction factors, which will differ from the one to 1 to 2 3 4 5 locations because the velocity distributions are the different.

So, as the velocity difference are there, so you will have a $B_1$, $B_2$, $B_3$, $B_4$, $B_5$ will be the difference because the momentum flux correction factors, it depends upon the velocity distribution. If when you have velocity distribution uniform, the beta one will be the one but if its not uniform, the beta one value will be the different. So, the basically if you look it to apply this concept, we use velocity distribution as a, considering the velocity distribution.

So we consider momentum flux correction factors, we have mass rate come into the velocity

which is the momentum flux rate that what we are summing it how much in and how much going out from this. Incoming momentum flux and outgoing momentum flux. As you will know from factorically, so outgoing momentum flux will be the positive and incoming momentum flux will be the negative. So, the rate of change of this net out flux of this momentum flux will be the force component.

The sum of the force component to what this control volume that what will be the net out flux of momentum fluxes through these control surface. So, in this case, we have three outlets, two inlets. So for the two inlets, we can compute the momentum flux from 1, 2. Similar we can compute the momentum flux, which is going out 3, 4, 5 we can compute it. Also we can consider, the momentum flux correction factors, which will be the different for the different inflow or outflow flow distributions, velocity distributions concerned.

So, we have considered that fact that there is a velocity distributions variability is there. Mass flux lean and out differently will be there. So considering this, in these control volume, fixed control volume, what will be the net force which will be acting on these will be considered as a steady problem, then you will have these conditions, okay. You can easily find out if it is a steady problem, you will have a definitely the net mass influx, in should equal to the net mass influx going out from this, that should be we get.

So, whatever the sum of the mass influx is coming it, that should be equal to some of the mass outflux is going out from this control volume, then we have steady problems. So, this is the conservation of mass equations and this is the linear momentum equations for the steady flow having multiple inlet and the outlet.

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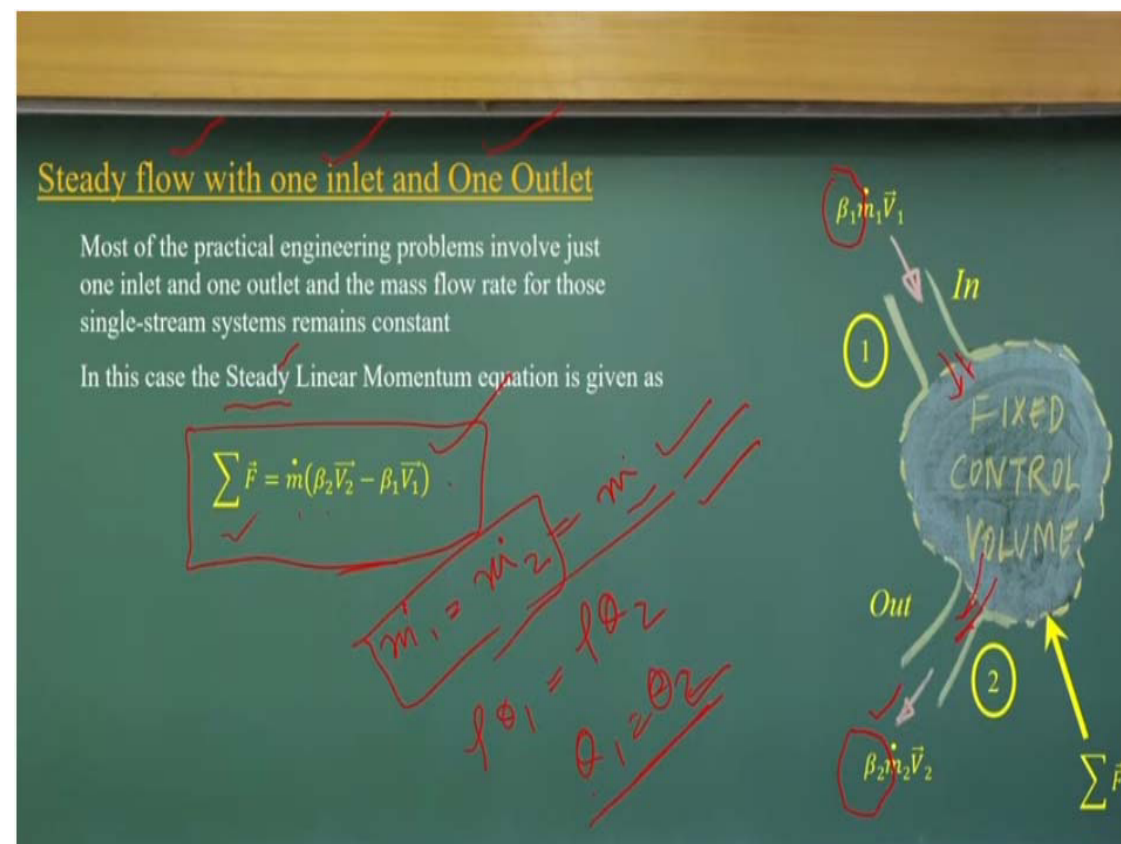
Now, let us go for another simplifications. That in a steady flow conditions, okay and only one outlet one inlet, this is very simplified case. See when you have one inlet and one outlet, if your mass influx is $\dot{m}_1$ is inflow is coming into that, your mass influx out will be $\dot{m}_2$, that should be equal. That means, $\rho Q_1 = \rho Q_2$ will be equal that means Q1 = Q2, that is a very basic equations what we get it, if we apply the conservations of mass.

So, if mass influx, what is coming if that equal to mass outflux, that what we can designate as a mass influx rate and then, the sum of the force acting on this as a steady conditions what will happen it, that some of force equal to the mass will come out $\beta_2$ P$_2$ – $\beta_1$ P$_1$. So, outlet inlet momentum flux what we are getting it with a correction factors of $\beta_1$ and $\beta_2$ we have use it to compute the momentum flux, what is coming into the control volume, what is going out from this control volume. The net momentum flux, that what will be the force component.

$$\sum \vec{F} = \dot{m}\left(\beta_2 \overrightarrow{V_2} - \beta_1 \overrightarrow{V_1}\right)$$

That is the equations what you get. So, these simplifications what we are doing it, that sometimes you can directly use it, these equations instead of to use Reynolds transport theorems, then you simplify it, then you come it. Instead of that, if you have solving a steady flow problem with one inlet and one outlet, to find out what is a force is acting in that, you can directly substitute these equations as a linear momentum equations and these equations you can use as mass conservation equation, which is the Q1 = Q2. That is very simple way we can do it.

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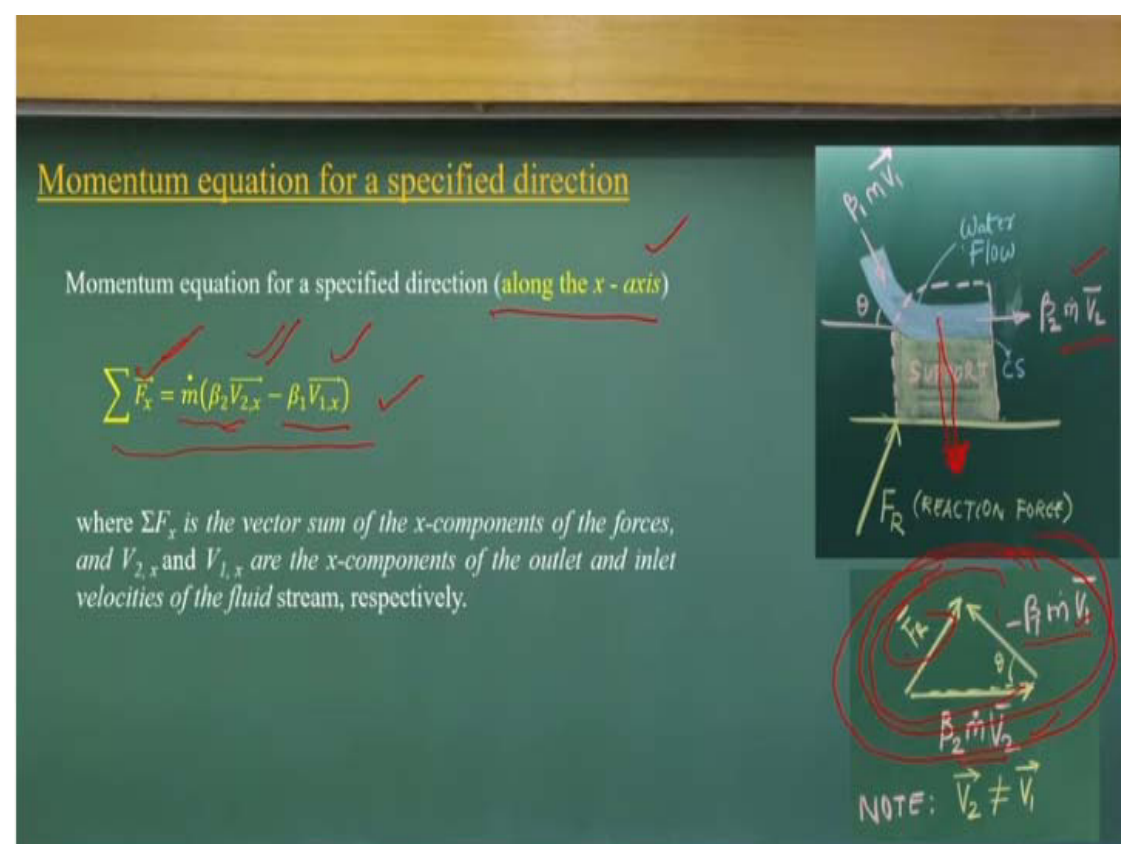
Now, there are the problems where is come it, very interestingly that we can apply the momentum equations as you can go it, this momentum equation is a vector equation, it has a three components in scalar directions, X component, Y component, Z component. Many of the time, we result at specific directions, in this case is X directions to find out the force components. That is what we try to look it, what will be the force is happening it, in the X directions only.

$$\sum \overrightarrow{F_x} = \dot{m}\left(\beta_2 \overrightarrow{V_{2,x}} - \beta_1 \overrightarrow{V_{1,x}}\right)$$

So, we can use it only the X directions component part, since there is, this problem if you take it, there is a water jet is coming in and going out, there is a change in the directions of the hot water jet okay. And there is a support system for that, what will be the force acting on this, that is what the problems. That means there is a water that is coming which is making an angle $\theta$, then making a turn it then becomes a horizontal water jet. So, because of the change of the momentum flux directions, what is the amount of force is acting on that.

If I am resumed it only this X directions component, then what I will do it, I will find out the momentum flux in X direction, not the Y or Z direction, because I can have a three equations. Here, we have a very good simplification that, if I am to compute the force component only the X directions, if I can see that the control volume the slide this, I know it the force, the gravity force act in the Z directions. So that force component will not commit. So, this is what quite simplifications we supposed to do it when you consider only the X direction.

The force component of X directions, you know it the force acting on the, either in Y or the Z

direction, that may not have any components. So, that way you can result the force component as a scalar component only X directions. So you compute the momentum flux what coming in the X direction momentum flux, similar way what is a momentum flux outgoing, that what we compute it. Sometimes you can exactly a vector method we can prove it if you do a vectorically.

The momentum flux in and out that can vectorically, that what is the momentum flux is a force components, so you can result this force component and the resultant force you can get. That is what we can simplify, you can solve this directly that these are the momentum flux as equivalent to force what is coming onto this control volume, going out from this control volume, you know these two force balance with a resultant forces.

That what vectorical you can find out what will be the $F_R$ value, that what will be the reaction forces. So, this way we can also simplify either considering this control volume or writing the force factors due to the momentum blocks in and the out, from there we can get the result and force components and we can get it the what is the x exponent force component. So note it that, B2$\beta_2$ should not equal equal to P$_1$, if it is that, so that because net force will be the 0.

So okay, there is no change in the velocity, directions or the magnitudes then there is no force which is going to act on this.

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Now if you look it there another set of the problems we solve it, where there is no external forces is there, there is no external forces is there, very simple cases. So, in that case, if I use

a linear momentum equations, the sum of the force will be the zero.

$$0 = \frac{d(m\vec{V})_{cv}}{dt} + \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$$

So what will happen it, that rate of change of momentum on a control volume that is what is here, that is which is equal to the difference between the rate of the incoming and outgoing momentum flux rates in absence of external forces.

If there is a, you consider a control volume, where we visualize conceptually that there is no external forces acting it. So, that problem is quite simplified, because if you apply the linear momentum equations, the net force is equal to zero. So zero what we have applied. So, problems becomes rate of change of momentum within the control volume is equal to the difference between the rate of incoming, outgoing momentum flux in absence of forces.

So, we have these three components with these if you can simplify it is a momentum rate what it changes is that what will be the as equivalent is mass of control volume and the acceleration component, it is just like a rigid body is moving it, okay. So, that what will be the force is moving it MD, that what will be, that force will be equate with the difference between the momentum flux in and the out. That the conditions will come it.

$$\frac{d(m\vec{V})_{cv}}{dt} = m_{cv}\frac{d\vec{V}_{cv}}{dt} = (m\vec{a})_{cv} = m_{cv}\vec{a}$$

The momentum flux in and out will be there, the sign convention if you look it, when you have a positive and negative, just have the difference equations of we are equating it. So, you can find out in a systems net flux, momentum flux is what is going on in and out, that what will be mass of bodies, M body and has accelerations that what will be equated that. If there is no external force acting on that or net thrusting force will be fixed mass body will be that much,.

$$\vec{F}_{thrust} = \vec{F}_{body} = m_{body}\vec{a} = \sum_{in} \beta \dot{m}\vec{V} - \sum_{out} \beta \dot{m}\vec{V}$$

This mostly we use it this equations for when you were launching the rockets, the net momentum flux what will you get it, the accelerator to the body. That the problems we solve it, I will solve one of the problems based on this concept.

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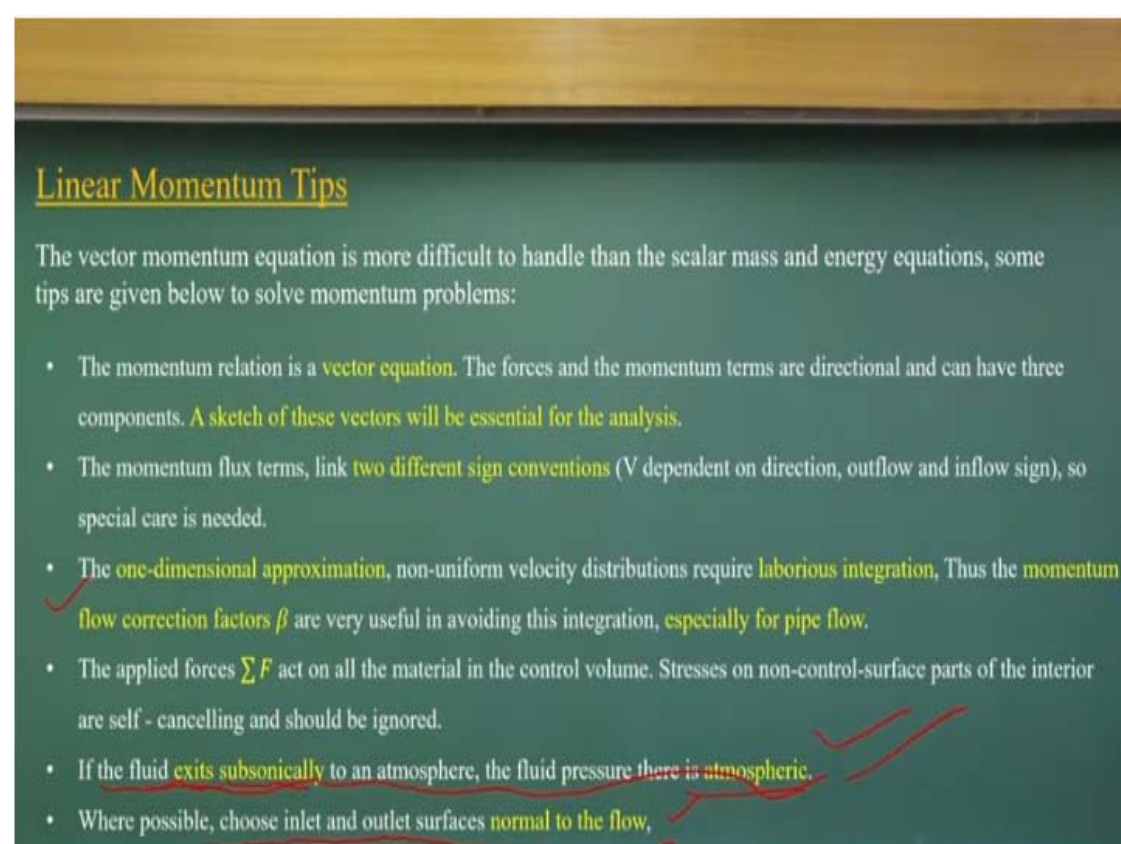
Now comes back to the, what are the tips are there, when you apply the linear momentum equation. First thing is that do remember is momentum equations is a vector equation, it has a three scalar components, it has the directions. You can write it in X direction equations, Y direction equation and the Z direction equation. You can write it in the three directions. So, many of the times, you do not result into three directions, we result the equations only one direction to solve a particular problem, okay.

But we can resolve this momentum relation in three directions X, Y, Z. So, when you solve this, any linear momentum the questions please case the vector components, then you visualize the fluid flow problems and then you solve it. Because it is a vector equation, you try to apply as a vector equation. Second is that we always compute the momentum flux terms, in flux and out flux terms. You always look it, what is a velocity direction. Whether it is a inflow or the outflow.

What will be the conditions when you have a dot product or the scalar product of the velocity and normal to the control surface, whether it will be a positive sign or a negative sign. That what always you visualize or always try to draw the velocity diagram first. Then, you have a control surface, you try to find out, if I multiply the scalar product of velocity and the normal vector to the surface, whether it will be the positive sign or the negative sign. That means if $\theta = 0$ between the velocity factors and the normal vectors.

Then we have cos theta equal to the 1. Or if you have a $\theta = 5°$, the $cos 5$ will be the -1. So, that what you try to look at that, what is the directions of velocity vectors, what is the directions

of the normal vectors to the control surface? If I do scalar product of the velocity and the normal vectors, what is the side, whether either positive or negative side. Many of the time, the student do these mistakes that they have a confused that whether you have to use a positive or negative sign.

Please find, try to understand it you are making a scalar product of velocity vectors and the normal vector. So, if you know what is the angle between these two, if $\theta = 0$ or $\theta = 5°$, you can find out whether it is a positive or the negative, this is very simple thing. But you guage the velocity factors, guage the normal factors then you apply that. And one of the things what we always discuss is that whenever you take a problems, the velocity distributions is not uniform.

Any real life problems, velocity distributions are not uniform, but we simplified it, make use of momentum flux correction factor. which considered the, because of non-uniform velocity distributions, what could be the correction factor to compute momentum flux through a control surface? So, that way, we always try to look it that, there is always a momentum flux correction factor will be there, if there is some velocity distributions is not uniform. For a real life problems velocity distribution cannot be uniform.

And that is the conditions we always have with the beta value or momentum flow correction factors. So, in a problem if the momentum flux correction factor is not given it, you can assume it or highlight it that, what you have done it that you have assume it that momentum correction flow corrections factor is 1 or unit file. That is what you have to consider, it is very important things about momentum flux correction factors and students who visualize that, there is a flow distribution the velocity distribution.

That what is considered when you use the momentum flux correction factors. And second things what is there, this applied force acting all the material in the control volume, we do not bother about inside the control volume, how the no control surface, the force acting part, that what self-canceling each other's, that what we do not consider it. We talked about over the control surface, what are the forces is acting it, that what we consider it.

And the last one is what is that the fluid when it exists subsonically, that means flow is subsonic flow, Mach number is less than 1. In that case, if flow exits to atmospheric conditions, you can

always consider the fluid pressure is atmospheric pressures. This is well known assumptions what we do it and it is quite valid that whenever the fluid exits subsonically, that means the flow is subsonic level, that means the flow is less than 1 to an atmospheric.

At that point, we can assume it, the pressure is equal to the atmospheric pressure, the pressure is equal to the atmospheric pressure. Similar way, as I said it earlier also, you always choose the inlet outlet, which will be normal to the flow condition. That is what I said it earlier this the scalar product between the velocity vector and normal vector to the control surface that should have a either $\theta = 0$ or $\theta = 5°$. So it is very simplified, we can solve the problems.

Otherwise, it can be done it with a scalar product of these two factors, it is not that difficult, but it is a laborious, maybe you need more time to solve the problem as compared to if you take a appropriate control volume, all the control surface. Then you can solve the problem easily. Let me repeat these things what I have talked to you, which is very important when you apply the linear momentum equation is that, the linear momentum equations what we get it is a vector equation.

It has three scalar components, X directions, Y directions, and Z directions. Many of the times when you solve our problems, you just apply one only one moment equations in one direction. You neglect other directions or that direction not necessary. But we remember the momentum equations is a vector equation. It has a three scalar directions equations we can write it. Second things which is again I am to repeat it that always you do not compete on the sign convention.

What time the momentum flux will be positive or the momentum flux will be the negative. Just look it that what will be the scalar product of the velocity vectors and normal to the control surface. Is it $\theta = 0$ or $\theta = 5°$. That way it will be define you that whether you will be positive fill or the negative fill. So, do not confuse that why this sometimes we consider momentum flux is positive, sometimes momentum flux is negative. So, these because of the scalar product.

The sign of the scalar product what you have consider it, between velocity and the normal vector to the control surface. The fourth one what again I am going to repeat it any real life problems the velocity distribution is not uniform. There is always a velocity distributions as they are from the pre surface to the surface of contact. There will be a velocity distribution. What we do it, we do not consider, always you do not consider integrations of that.

Instead of that we consider distortions of velocities, compute the momentum flux correction factors for that type of flow velocity distributions and that correction factor we use and the average velocity to compute momentum flux. So momentum flux correction factor and average velocity we have used to compute momentum flux. What is going through a control surface, where you have the velocity distribution, it is not uniform. If you uniform it, velocity is the same throughout this control surface.

Beta becomes 1 and last two points, what I am talking that many of the cases what you have considered this flow exits into the atmospheric at the subsonic flow levels, that means flow Mach number is less than 1. In that case, you can consider the pressure is equal to the atmospheric pressure. And the last point already I discuss is that, we always choose the control surface such a way that it should have a normal vector and the velocity vector should be having an angle of $\theta = 0$ or $\theta = 5°$.

That is the main tips what we need to do it before applying the linear momentum equations. We remember it, it is very easy to solve any complex problems if appropriate, linear control volume we consider and you have assumptions and all things you clearly noted. Looking that, I know it this problem what I am going to solve with these four examples, can be solved directly putting the momentum equations. But I do not follow that.

I follow very systematically the first flow classifications, second I do appropriate control volume, skies pressure diagram, force diagrams, then velocity diagrams, then apply the continuity equations, finally I use the momentum equations. So, if you look it that, whenever you get a problem, do not be excited that to apply the linear momentum equations as of you got the equation, simplify questions. Instead of that, please follow these procedures.

That first you do a classifications of fluid flow problems, what type of problems you have. Second, you try to stage the control volumes, which is appropriate control volumes and the control surface. Third, you look it, what could be the pressure distributions. Fourth what could be the velocity distributions or momentum flux correction factors. There you go for, applying the mass conservation equation and linear momentum equation.

Please follow these procedures, then if you follow these procedures, I think doing the error

during the solving the problems that what will be nullified it. Otherwise, if you apply the linear momentum equations without a proper control volume, it is expected that you will do a mistakes compared to if you follow appropriate control volume, all the assumptions are clearly highlighted, then you can solve the problems and that is what you will have a chance to do the error will be nullified as compared to solve directly using linear momentum.

To demonstrate that, I am just going to solve these four problems and you look it, how I am doing the assumptions one by one, then I am applying the mass conservation equations and the linear momentum equations, not directly applying the mass conservations or linear momentum equations. Yes let us solve four examples considering the concept what I said earlier. First examples I have taken it that a linear flow is passing through a very long straight round pipe with external velocity components.

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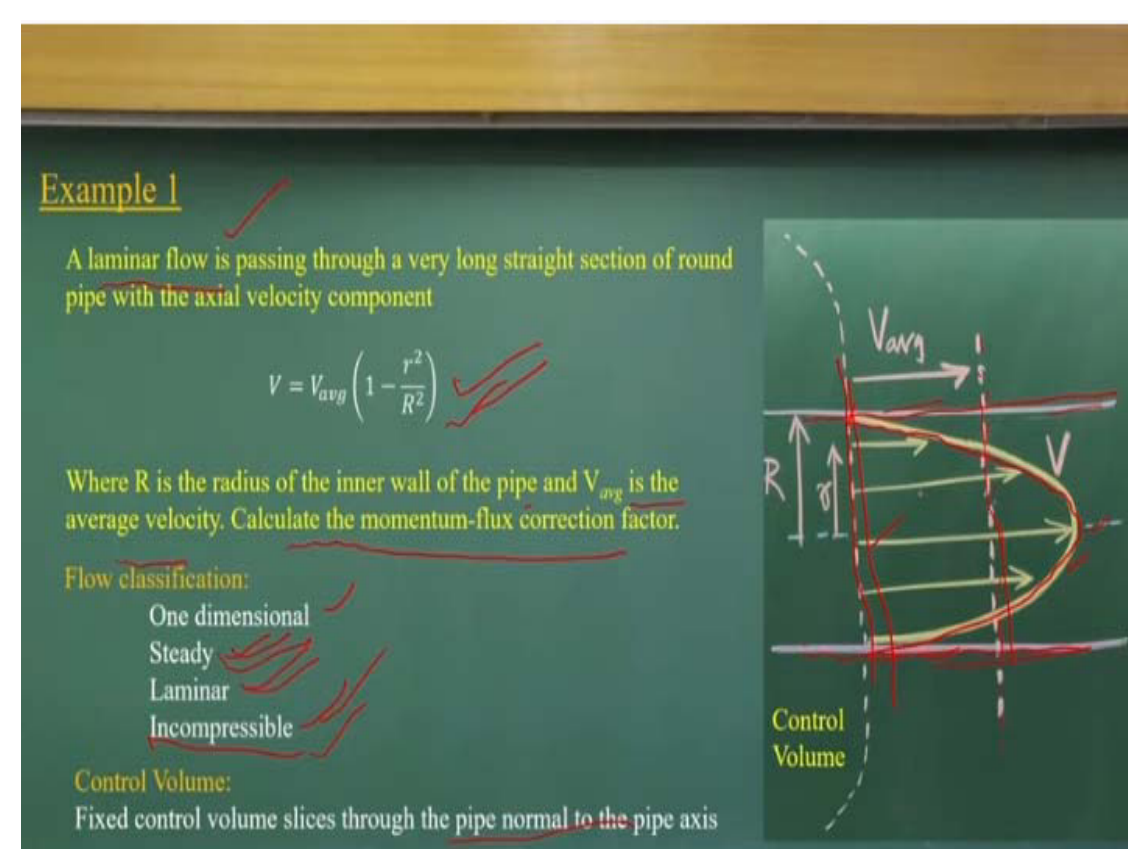


[A laminar flow is passing through a very long straight section of round pipe with the axial velocity component

$$V = V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

Where R is the radius of the inner wall of the pipe and $V_{avg}$ is the average velocity. Calculate the momentum-flux correction factor. ]

So velocity components is given it, how it varies it, okay. This is the five, okay. If R is a radius of inner wall of the pipe, $V_{avg}$ is the average velocity, then calculate the momentum flux correction factors. This is very simple problems. If you look it, you first draw the sketch. The

sketch says that you have the pipe, okay and there is a velocity distribution, which is equal to r square.  So that way, the velocity is maximum at this point, R that what will be the velocity becomes 0.

So, that way, you will have the velocity distribution what you would, approximately you could plot it, okay, what could be the velocity distribution.  And somewhere, this average velocity will come it like this, okay.  So this is the velocity distributions, the laminar flow through a long straight pipe, this is the tentative velocity distribution.  Now we are going to compute it, what could be the momentum flux correction factor, if this is the velocity distribution.

Flow classification:

       One dimensional

       Steady

       Laminar

       Incompressible

No doubt, there is no time component is there, so this is a steady flow. Flow is laminar, is already highlighted and we can assume it, in this case the pipe flow may not have the Mach number of more than 0.3, the flow can be considered as incompressible flow.  Here okay, will not consider this control volume, okay.  We can consider as a control volume and slicing the pipe normal.  Just you consider a control volume, where you just want to make it what will be the velocity distributions.

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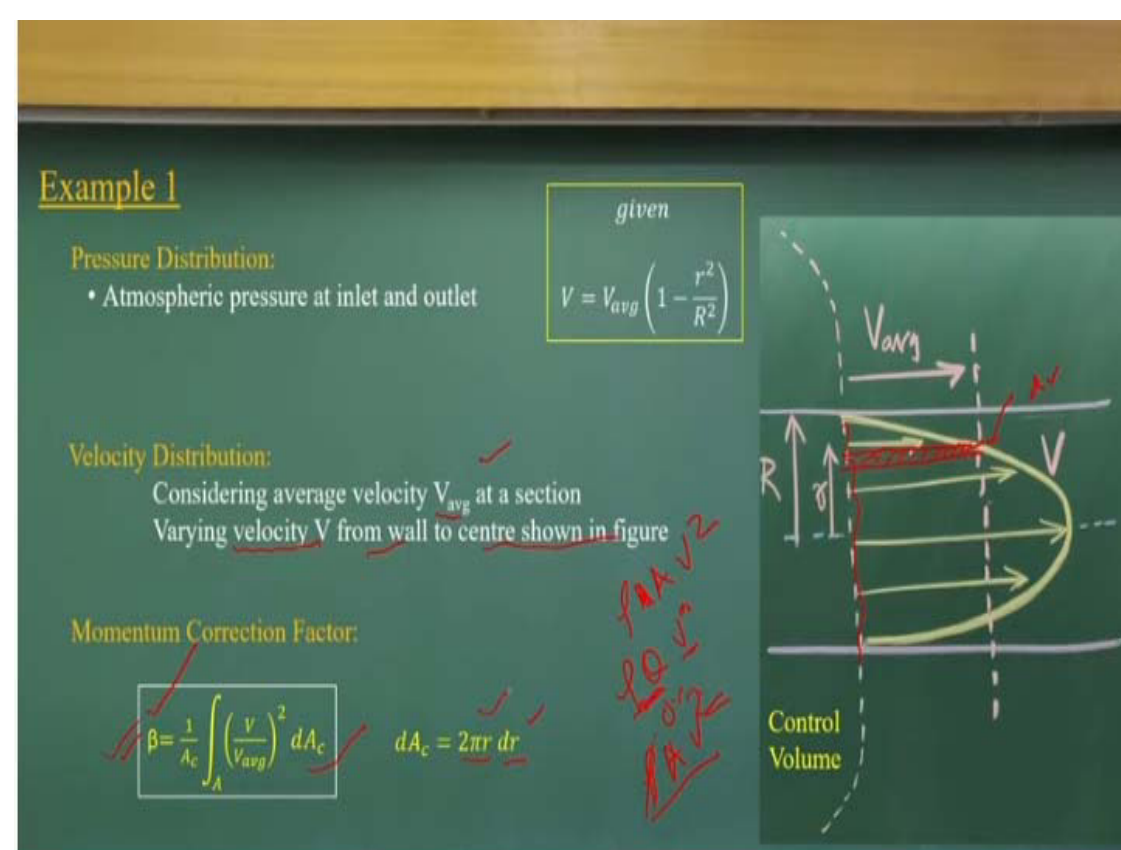


Now, we have these distributions like this and since I have the average velocity $V_{avg}$ at a

sections and the velocity V vary from wall to the centers as given in this figures. Then I am to compute the momentum flux correction factors, what is that? We need to compute the momentum flux. If I consider the velocity distribution, divide by the momentum flux. If I consider V = V average, the ratio between these two terms will give me the momentum flux correction factors, if you remember the definitions.

Pressure Distribution:

- Atmospheric pressure at inlet and outlet

Velocity Distribution:

Considering average velocity $V_{avg}$ at a section

Varying velocity V from wall to centre shown in figure

So, if you apply it, what is the momentum flux? $\rho AV^2$ or $\rho QV$, $\rho Q$ is my mass flux, V is the velocity, mass into velocity is the momentum flux. $\rho QV$ or $\rho AV^2$ is a momentum flux. Now, if $\rho$ is a constant, does not vary it, so it will be cancelled out it, then you will be $V^2$ by V average square by the $dA_c$. So please remember it, it is a very simple concept is that the momentum flux what, $\rho QV$ and if I substitute Q equal to AV, then $\rho AV^2$.

$$\beta = \frac{1}{A_c} \int_A \left(\frac{V}{V_{avg}}\right)^2 dA_c$$

$$dA_c = 2\pi r \, dr$$

That is the point what we will do it. If I have considered element area dr, the area will be the circumference into the dr, that what will the area, length into dr will be the area part.

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